Nonlinear Wave Loads and Structural Response

What matters and what doesn't

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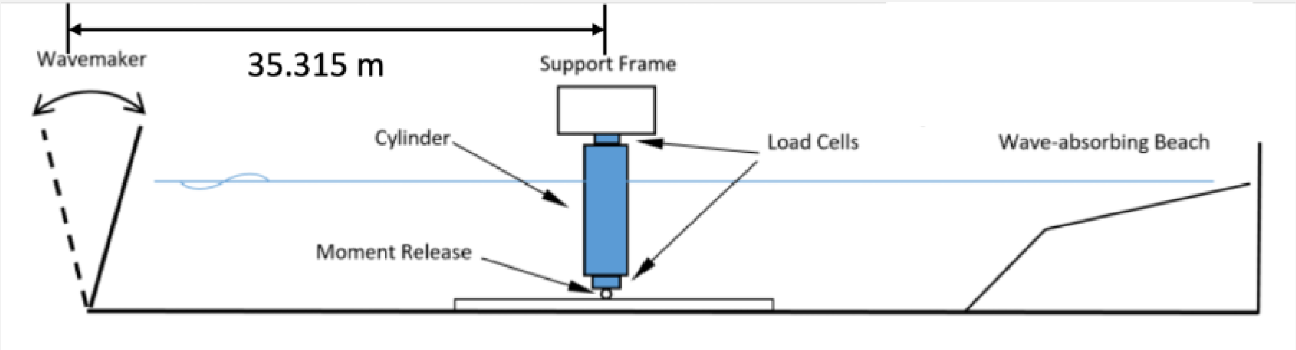
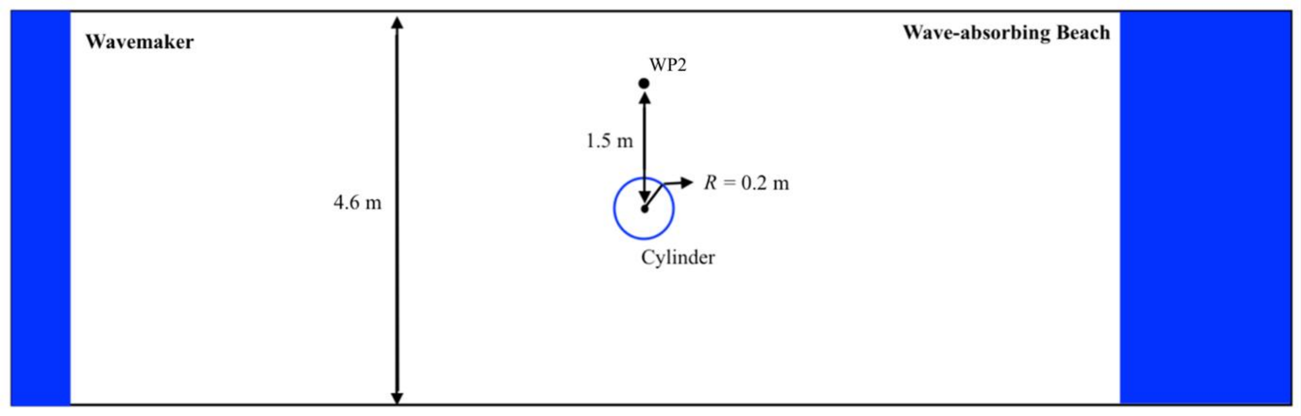
KHL Tank – Phase 2





- **Dimensions:**

76 m (Length) ×

4.6 m (Width) ×

2.5 m (Height)

- **Water Level:**

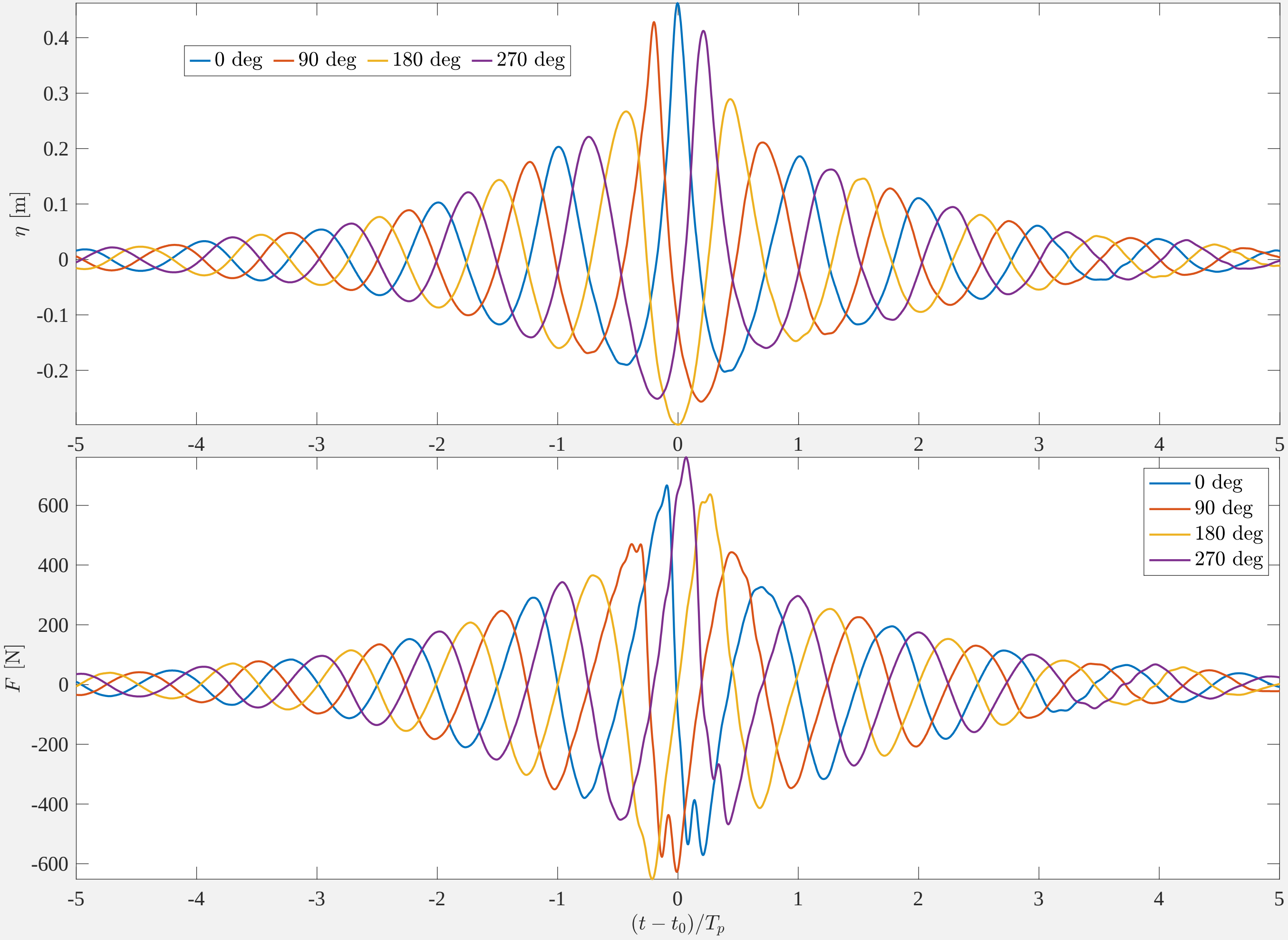
depth of 1.8 m

Wave Group Tests Parameters

Test	**A [m]**	**Tp [s]**	**kp·d**	**kp·A**	**kp·R**	**KC**
WG3	0.279	2.53	1.31	0.203	0.146	4.38
WG6	0.314	2.53	1.31	0.229	0.146	4.94
WG4	0.318	2.53	1.31	0.231	0.146	4.99
WG7	0.324	2.53	1.31	0.236	0.146	5.08
WG10	0.311	2.25	1.56	0.270	0.174	4.88
WG9	0.328	2.25	1.56	0.285	0.174	5.15

WG9: 4-phase runs (time histories)

- Steepest case
- Crest focusing on cylinder
- (η, F) are 90-deg out of phase
- Same linear wave envelope
- F more nonlinear (SLC, later)



Elevation & Force Structure
- Stokes-type expansions

$$\eta(t) = A\cos\phi + A^2(\mathcal{K}_{20} + \mathcal{K}_{22}\cos 2\phi) + A^3(\mathcal{K}_{31}\cos\phi + \mathcal{K}_{33}\cos 3\phi)$$
$$+ A^4(\mathcal{K}_{40} + \mathcal{K}_{42}\cos 2\phi + \mathcal{K}_{44}\cos 4\phi) + \mathcal{O}(A^5),$$

$$F(t) = Af_{11}\cos\psi + A^2\left[f_{20}\cos(-\varphi_{20}) + f_{22}\cos(2\psi - \varphi_{22})\right]$$
$$+ A^3\left[f_{31}\cos(\psi - \varphi_{31}) + f_{33}\cos(3\psi - \varphi_{33})\right]$$
$$+ A^4\left[f_{40}\cos(-\varphi_{40}) + f_{42}\cos(2\psi - \varphi_{42}) + f_{44}\cos(4\psi - \varphi_{44})\right] + \mathcal{O}(A^5)$$

- Elevation: $\phi = \omega t + \varepsilon_0$;
- Force: $\psi = \omega t + \epsilon_0$,
  φ_{mn} phase shift + amplitude coeff. – harder to model F

4-phase separation

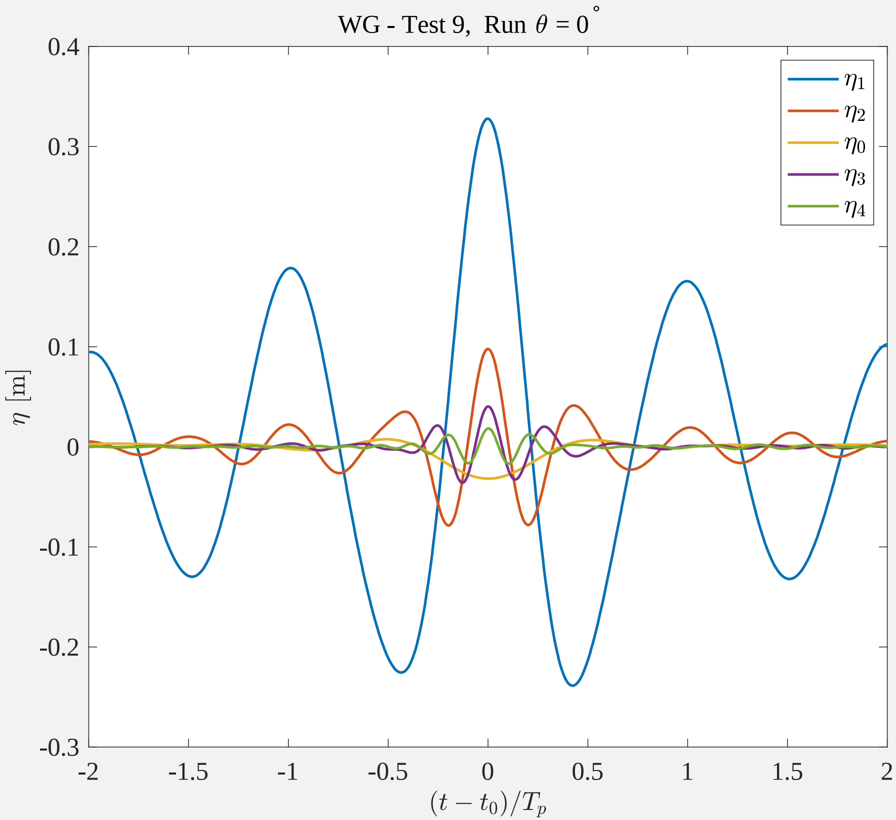
Given the four-phase runs for the nonlinear force, F, the separation is straightforward:

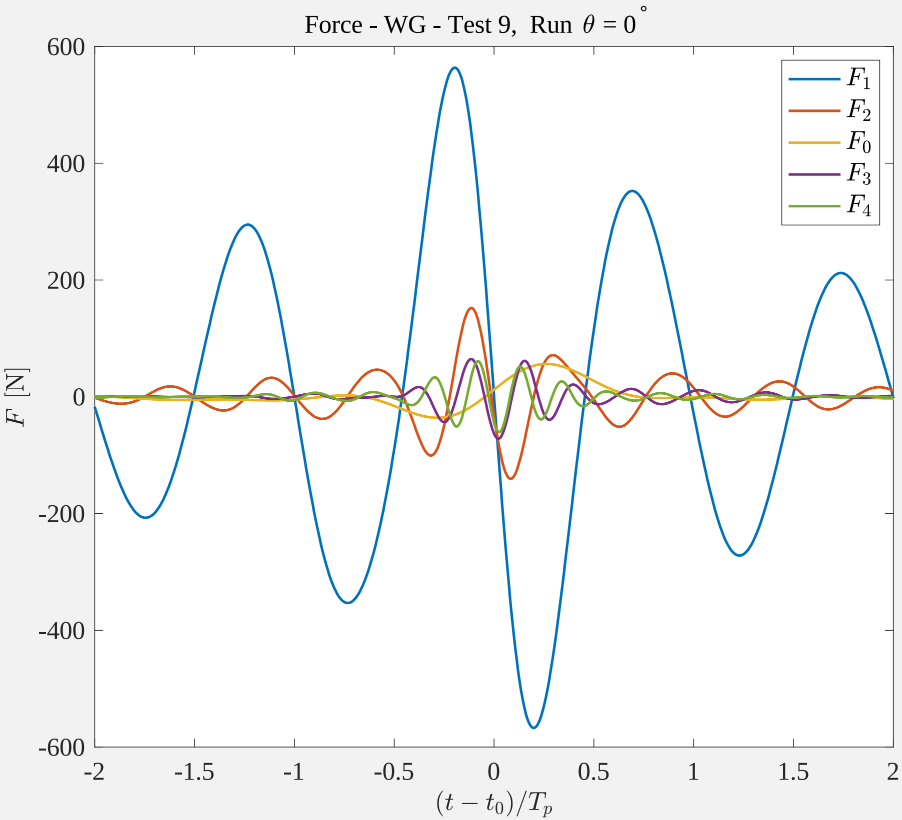
$$\frac{F^{(0)} - F^{(90\mathrm{H})} - F^{(180)} + F^{(270\mathrm{H})}}{4} = F_{11} + F_{31} + \mathcal{O}(A^5),$$
$$\frac{F^{(0)} - F^{(90)} + F^{(180)} - F^{(270)}}{4} = F_{22} + F_{42} + \mathcal{O}(A^6),$$
$$\frac{F^{(0)} + F^{(90\mathrm{H})} - F^{(180)} - F^{(270\mathrm{H})}}{4} = F_{33} + \mathcal{O}(A^5),$$
$$\frac{F^{(0)} + F^{(90)} + F^{(180)} + F^{(270)}}{4} = F_{20} + F_{40} + F_{44} + \mathcal{O}(A^6),$$

- H is the Hilbert transform
- Same calculations for η

Fitzgerald, C.J., Taylor, P.H., Eatock Taylor, R., Grice, J., Zang, J.
Phase manipulation and the harmonic components of ringing forces on a surface-piercing column (2014) Proc. Roy. Soc. A, 470 (2168), art. no. 20130847.

Elevation & Force Harmonics





All elevation harmonics in phase at focus point

All force harmonics close-to-zero crossing

Stokes-type Fitting of F Harmonics

- Calculations done in the frequency domain

$$F(t) = Af_{11}\cos\psi + A^2\left[f_{20}\cos(-\varphi_{20}) + f_{22}\cos(2\psi - \varphi_{22})\right]$$
$$+ A^3\left[f_{31}\cos(\psi - \varphi_{31}) + f_{33}\cos(3\psi - \varphi_{33})\right]$$
$$+ A^4\left[f_{40}\cos(-\varphi_{40}) + f_{42}\cos(2\psi - \varphi_{42}) + f_{44}\cos(4\psi - \varphi_{44})\right] + \mathcal{O}(A^5)$$

- $\psi = \omega t + \epsilon_0$: linear phase
- ϵ_0 : phase shift
- φ_n : 2nd phase shift

Amplitude coeff.

2nd Phase shift.

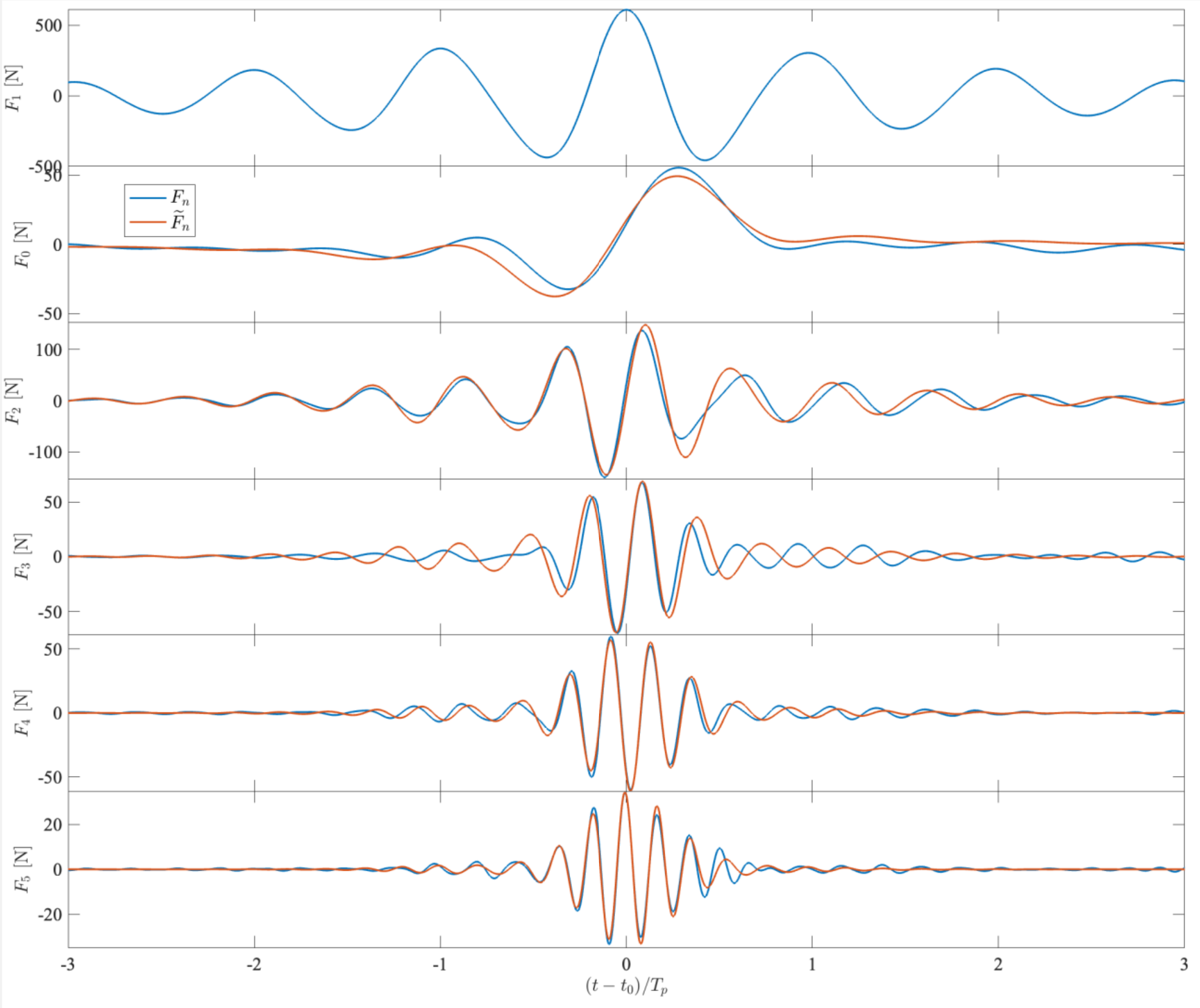
AIM:

$$\widetilde{F_n} = \Gamma_n \widehat{F_1}^{\,m} \cos(n\psi - \varphi_n)$$

- $\Gamma_n = \widehat{F_n} \times \widehat{F_1}^{\,-m}$
- φ_n from minimisation via FFT

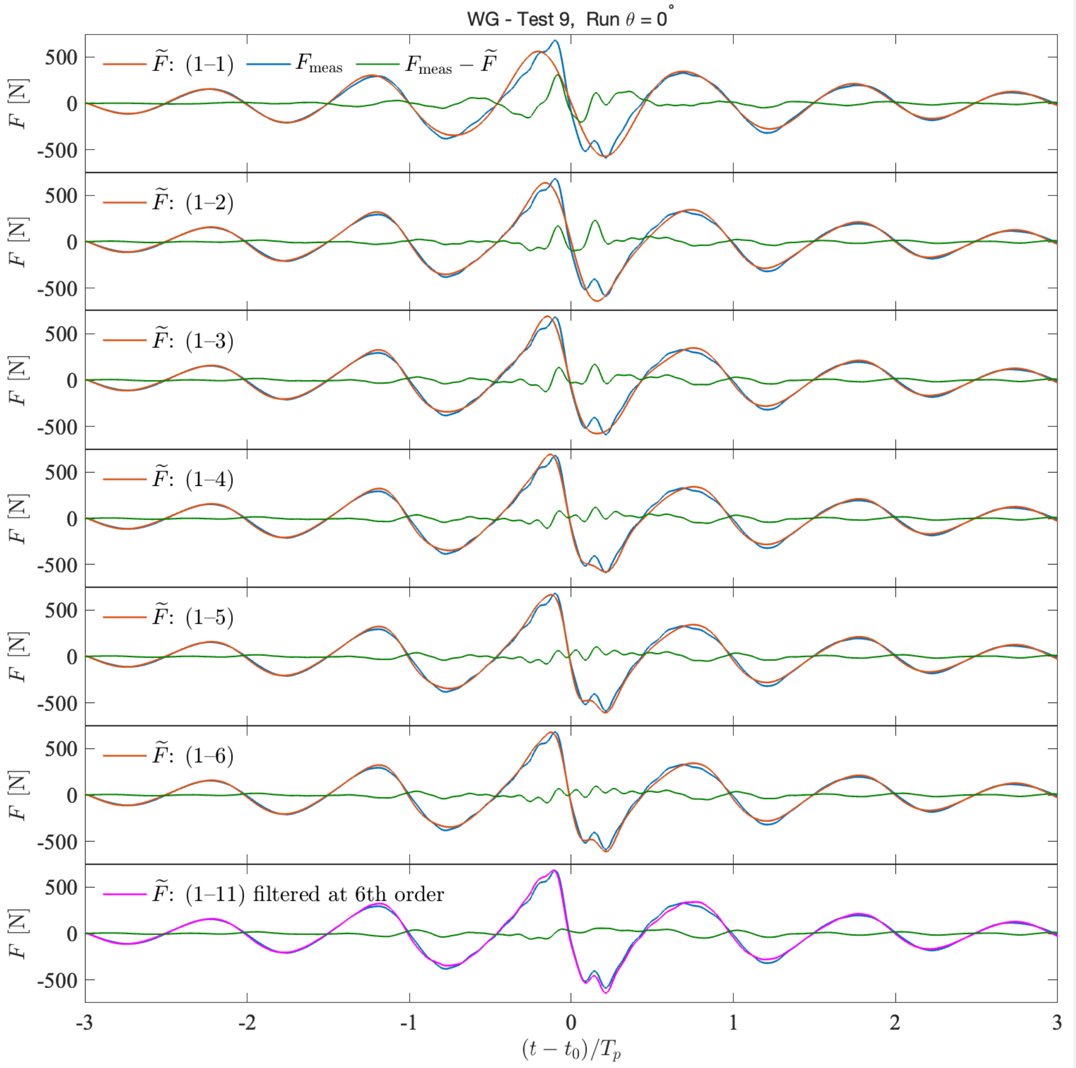
Reconstructing harmonics of total force F using powers of linear component F_1

Good agreement in general – main structure captured well



Summing reconstructed F harmonics

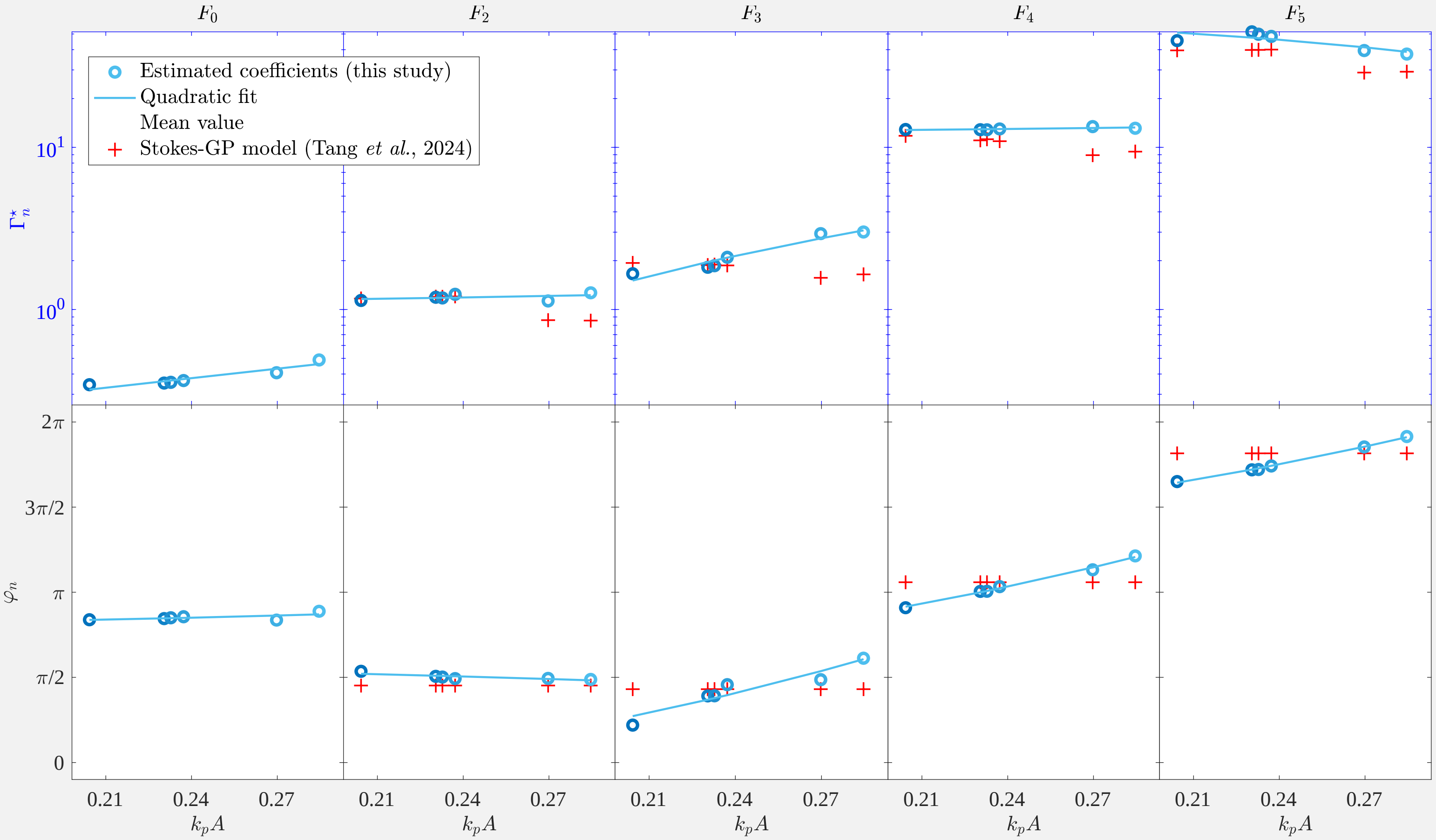
- Linear approx. is inadequate
- Improvement up to ~5th order – beyond only SLC improves
- Stokes-type expansion APPEARS to capture SLC



Fitting coefficients

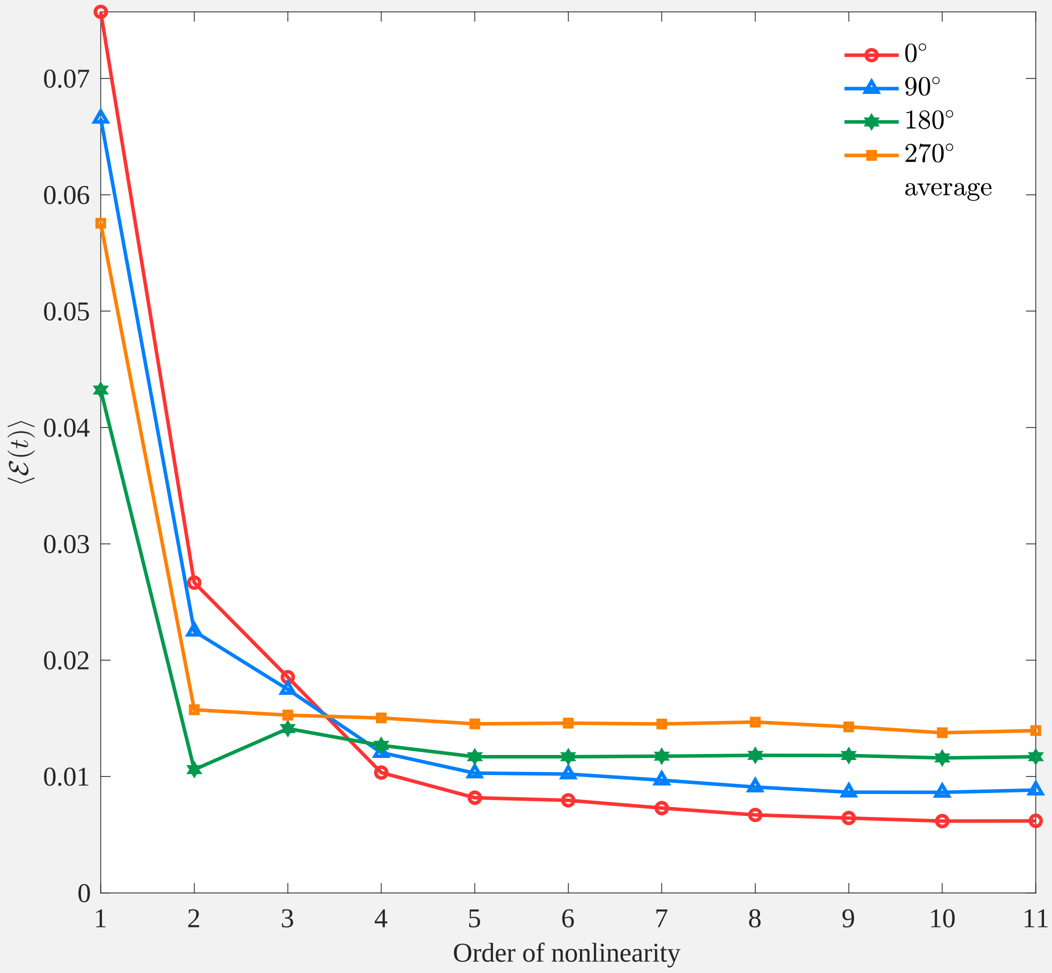
- Good match with previous works

- Phases (2^{nd} –5^{th}) match Mj et al. 2023

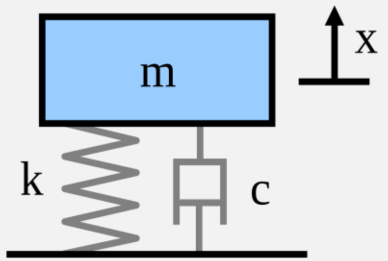


Is 5th order sufficient?

- MSE Error averaged over time
- YES, 5th order F models should be sufficient



Mass-spring-damper system



$\zeta = \frac{c}{2\sqrt{Mk}}$ damping ratio

$\omega_R = \sqrt{\frac{k}{M}}$ resonant freq.

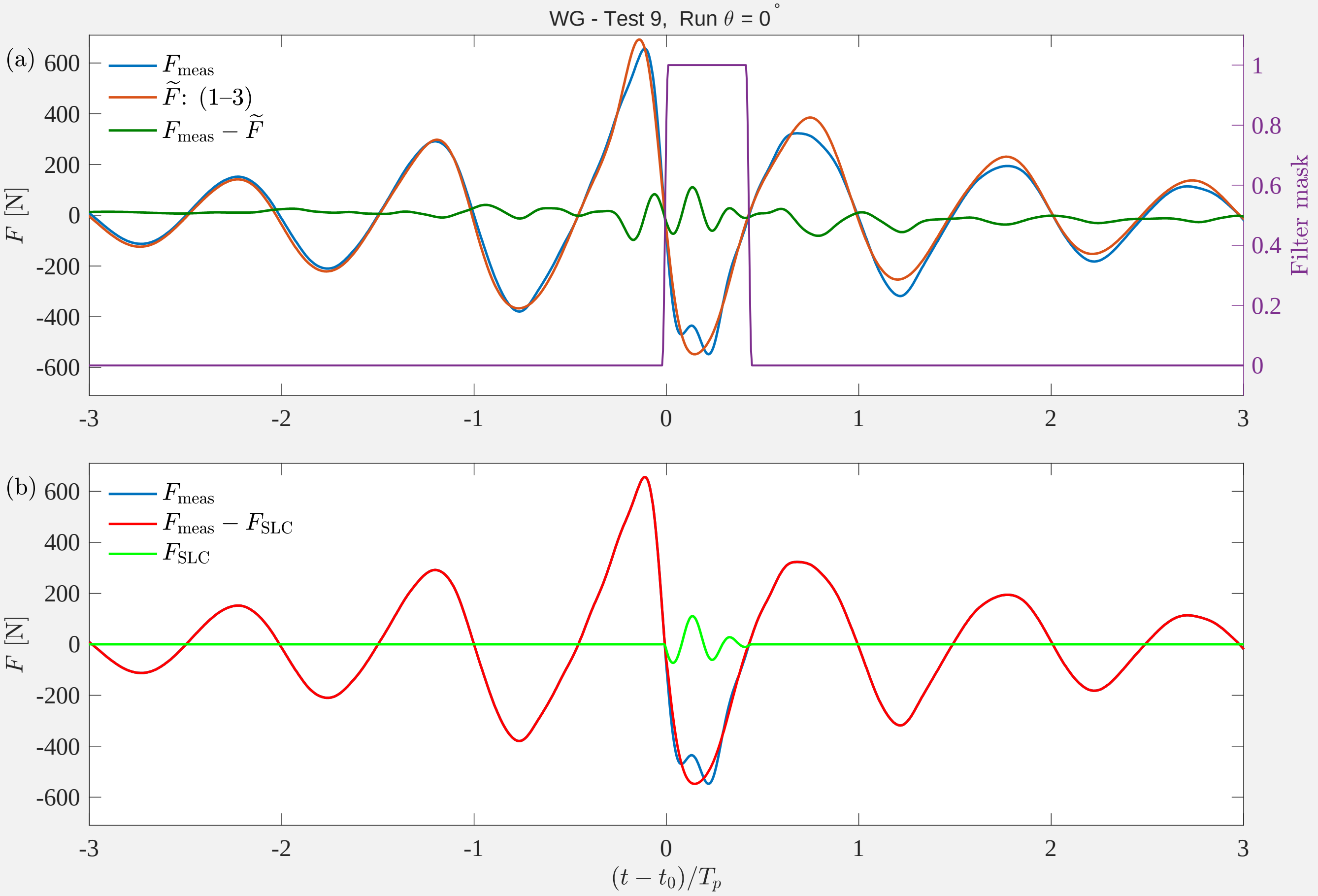
$$F(t) = m\ddot{x} + c\dot{x} + kx$$

$$F(\omega) = (k + ic\omega - m\omega^2)x(\omega)$$
$$= \left(1 + i\frac{c}{k}\omega - \frac{m}{k}\omega^2\right)kx(\omega)$$
$$= \left(1 + i2\zeta\frac{\omega}{\omega_R} - \frac{\omega^2}{\omega_R}\right)kx(\omega) = H(\omega)^{-1}kx(\omega)$$
$$= H(\omega)^{-1}X(\omega)$$

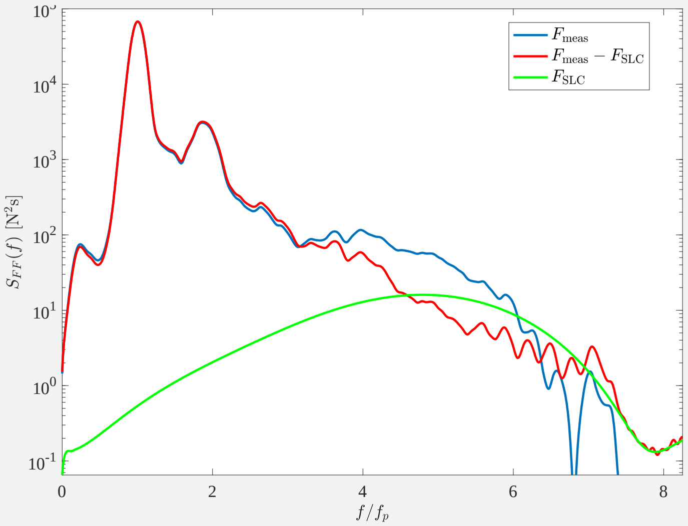
F is applied (measured)

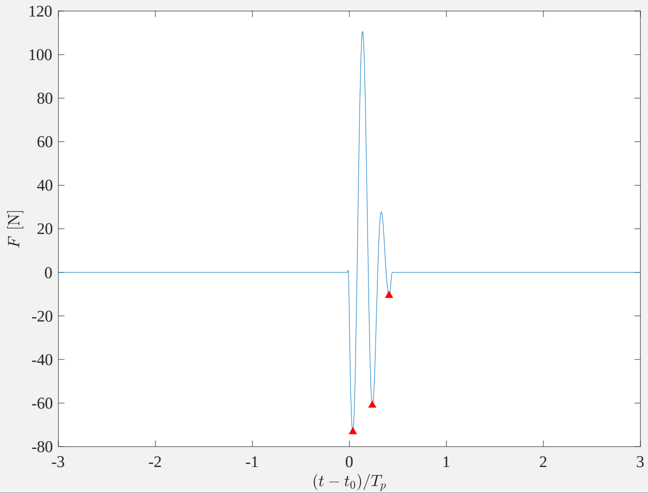
$X = kx$ is the response (unknown);

Secondary Load Cycle - Extraction

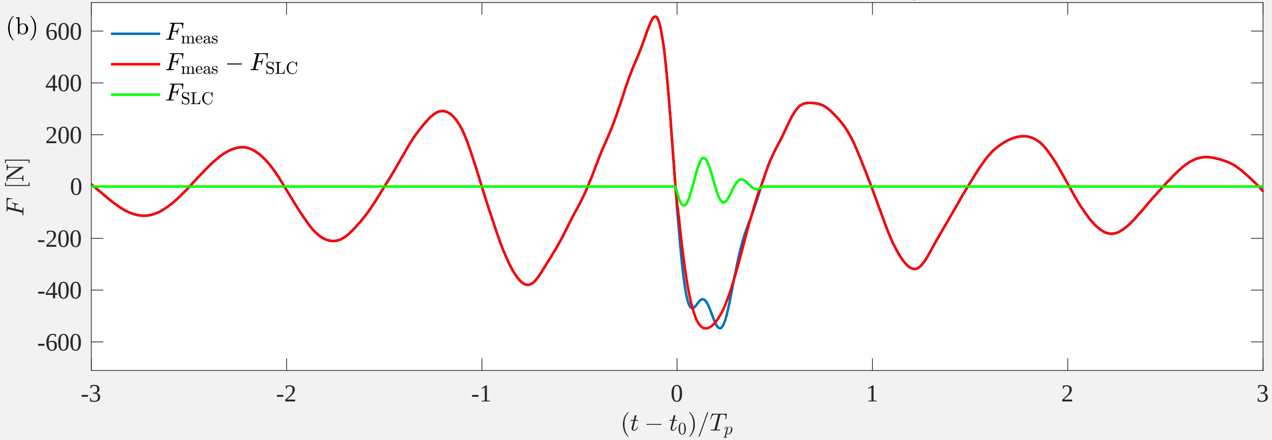


Secondary Load Cycle (SLC)



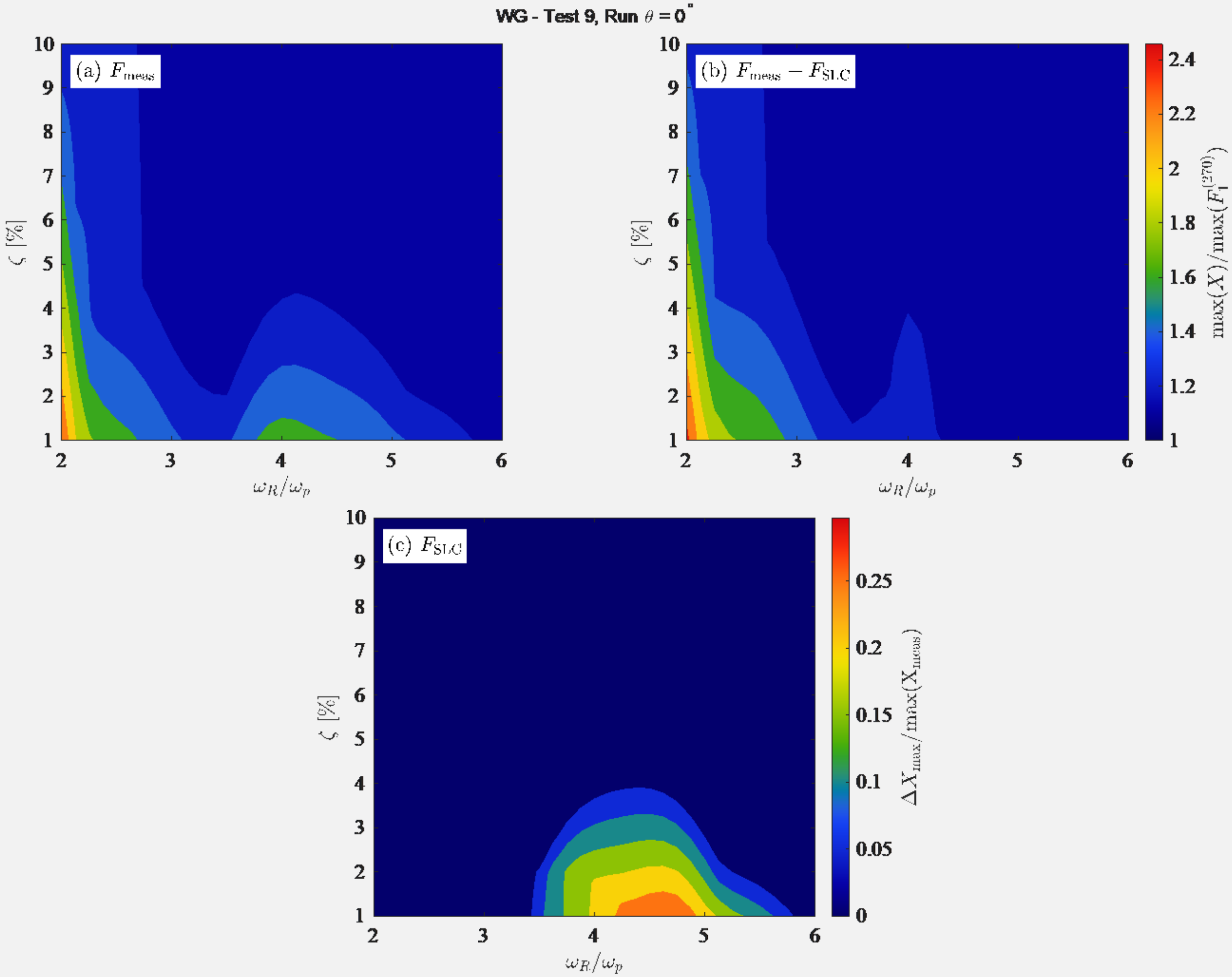


- T = 0.14 to 0.2Tp
- Freq. = 4.4 to 5fp



Dynamic response due to SLC

- $\hat{F}_{SLC}/\hat{F}_{meas}$ = 0.143
- X rel. response up to ~0.28
- SLC not important for $\zeta > 4\%$

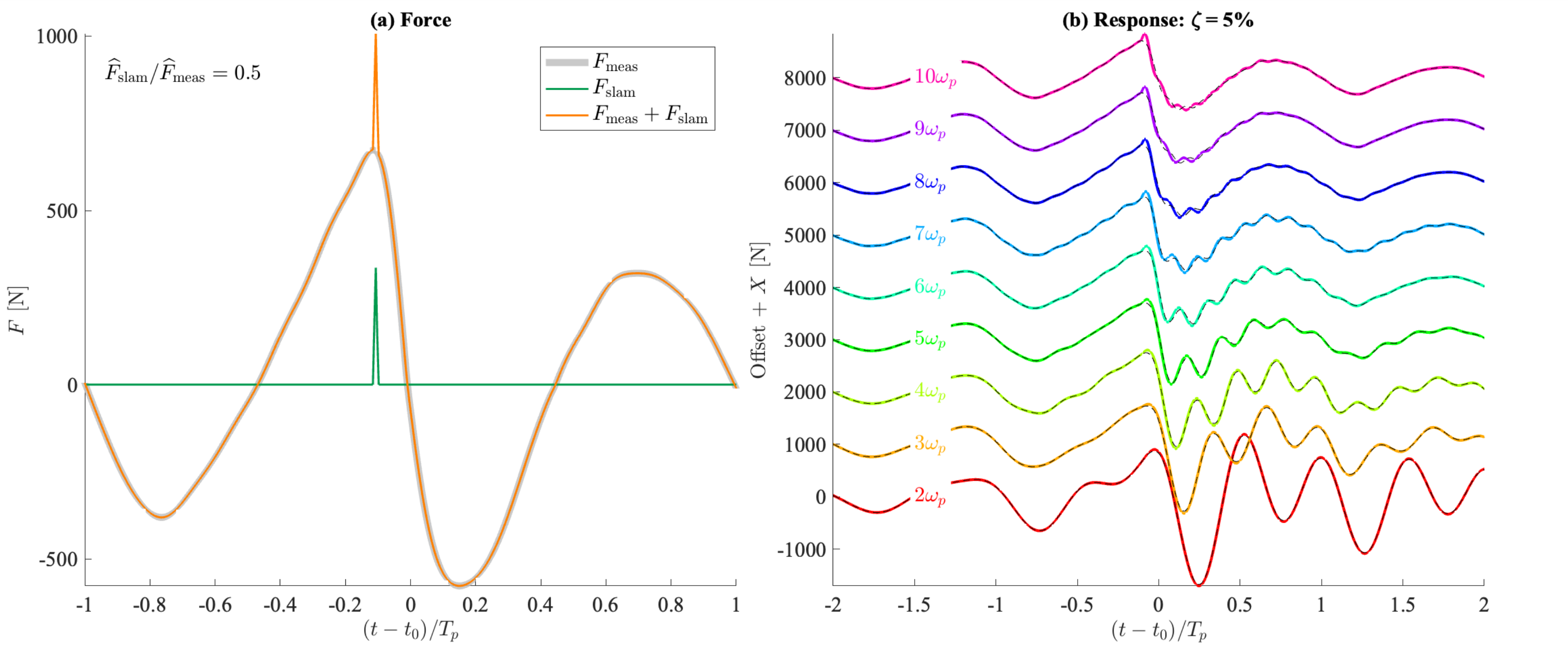


Slamming

- Paulsen (2019): $F_{slam}(t) \propto \sin^2(\pi \frac{t-t_{crest}}{T_S})$
- $T_s = \frac{13D}{32C_p} \sim \frac{R}{C_p}$ (slamming period)

- For WG9: Tp = 2.25 s, Ts = 0.0387 s
- Slam freq ~58 x wave freq

Response due to slamming



- Slam-related response is small (< 20% of measured X from F w/o slam);
- Slamming is NOT important for design

Conclusion

- Stokes-type fitting is adequate for structural behaviour

For overall response:

- Secondary load cycle is practically unimportant
- Slam loads appear to be relatively unimportant